The Rarita-Schwinger spin-$\frac{3}{2}$ equation in a nonuniform, central potential

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Abstract

The equations of motion for a massive spin-$\frac{3}{2}$ Rarita-Schwinger field in a finite-range, central, Lorentz scalar potential are developed. It is shown that the resulting density may not be everywhere positive definite.

I. INTRODUCTION

For many years there has been interest in studying spin-$\frac{3}{2}$ particles in as much detail as has been done for spin-$\frac{1}{2}$ particles. In the latter case, the Dirac equation has been shown to provide a reasonable starting basis for such studies [1], although of course the formalism of quantum field theory is needed for any detailed consideration. Jasiak and Szymacha [2] have attempted to use the Rarita-Schwinger equations for spin-$\frac{3}{2}$ particles in order to investigate the Δ resonance in the bag model. However, they restricted themselves to solving only a subset of the equations, and made no attempt to verify whether or not the wave function that they calculated was physically meaningful. In this paper, we address this question by explicitly considering the most general form for the wave function.

Attempts to quantize the spin-$\frac{3}{2}$ Rarita-Schwinger field in the presence of interactions of scalar and vector type are beset with problems [3]. For example, it is known that the Rarita-Schwinger field has noncausal modes of propagation [4]. Johnson and Sudarshan [5] found that for vector interactions, the canonical anti-commutator is not positive definite at

all spacetime points. Here we will restrict our considerations to the classical field equations.

Our work here is motivated by relativistic QHD models of Walecka and collaborators [1,6], which consider nucleons interacting with mean-field scalar and vector potentials by the Dirac equation. Our initial purpose in undertaking this work was to describe the Δ by an analogous relativistic equation. This is clearly important in order to obtain a full treatment of the electromagnetic response of the nucleus, particularly for deep inelastic electron scattering in the quasi-elastic regime and beyond [7]. An alternative approach, taken by Lin and Serot [8], is to treat the Δ dynamically as a $\pi - N$ resonance, thus avoiding the need for an explicit equation for the Δ. Here we treat the Δ as an additional degree of freedom, described by the relativistic spin-$\frac{3}{2}$ Rarita-Schwinger equation. This is analogous to the non-relativistic Δ-hole models.

II. FORMALISM FOR NONUNIFORM, CENTRAL SCALAR POTENTIALS

The free-particle Rarita-Schwinger equations $(i\,\partial\!\!\!/ - m)\Psi^\mu = 0$, and the auxiliary condition $\gamma_\mu \Psi^\mu = 0$, can both be derived from the Lagrangian

$$\mathcal{L}_{RS} = \overline{\Psi}_\mu(\partial\!\!\!/ + i\,m)\Psi^\mu - \frac{1}{3}\overline{\Psi}_\mu(\partial^\mu\Phi + \gamma^\mu\partial\cdot\Psi) + \frac{1}{3}\overline{\Psi}_\mu\gamma^\mu(\partial\!\!\!/ - i\,m)\Phi, \tag{1}$$

where $\partial\!\!\!/ \equiv \gamma_\mu\partial^\mu$, $\Phi \equiv \gamma_\mu\Psi^\mu$, and $\partial\cdot\Psi \equiv \partial_\mu\Psi^\mu$. The Euler-Lagrange equations of motion corresponding to $\mathcal{L}_{RS}$ are given by

$$(\partial\!\!\!/ + i\,m)\Psi^\mu - \frac{1}{3}(\partial^\mu\Phi + \gamma^\mu\partial\cdot\Psi) + \frac{1}{3}\gamma^\mu(\partial\!\!\!/ - i\,m)\Phi = 0. \tag{2}$$

Contracting this result with γ_μ and ∂_μ results in the subsidiary conditions $\Phi = 0$, and $\partial\cdot\Psi = 0$. Hence Eq. (2) reduces to the usual Rarita-Schwinger equations. Note that Eq. (1) is not the most general form for $\mathcal{L}_{RS}$. In fact, there is a two-parameter Lagrangian that leads to exactly the same free equations of motion regardless of the values of the parameters.

The most general form of bilinear, non-derivative couplings of a massive Rarita-Schwinger field was investigated by Hagen and Singh [3]. Here we are interested in the simplest case of

a Lorentz scalar potential. In order to introduce this interaction potential into the Rarita-Schwinger formalism, we follow the approach used for the Dirac equation (and applied to the spin-$\frac{3}{2}$ case by Jasiak and Szymacha [2]) and replace the constant mass $m = m_0$ in the Lagrangian and resulting Euler-Lagrange equations by a mass $m(r) = m_0 - U(r)$, which is a spherically symmetric function of radius r. This approach is consistent with that of Hagen and Singh [3]. The scalar potential $U(r)$ is of finite range, and vanishes at sufficiently large values of r.

The subsidiary conditions that were derived from Eq. (2) now become

$$\partial \cdot \Psi + \frac{1}{m}\frac{dm}{dr}\,\hat{\boldsymbol{r}} \cdot \boldsymbol{\Psi} = 0, \tag{3}$$

$$2\partial \cdot \Psi - i\,m\Phi = 0. \tag{4}$$

The Euler-Lagrange equations can be written in the forms:

$$(\not{\partial} + i\,m)(\Psi^0 - \frac{1}{3}\gamma^0\Phi) + \frac{1}{6}(2\partial^0 - i\,m\gamma^0)\Phi = 0, \tag{5}$$

$$\hat{\boldsymbol{r}} \cdot (\not{\partial} + i\,m)\boldsymbol{\Psi} + \frac{1}{3}\frac{\partial \Phi}{\partial r} - \frac{1}{3}\gamma^r \partial \cdot \Psi + \frac{1}{3}\gamma^r(\not{\partial} - i\,m)\Phi = 0, \tag{6}$$

where $\gamma^r \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{\gamma}$. These two equations, along with the two subsidiary conditions (3) and (4), replace the original four Rarita-Schwinger equations.

Using the usual Dirac representation of the gamma matrices γ^μ, it follows that $\not{\partial} = \gamma^0\partial^0 + \gamma^r\left(\frac{\partial}{\partial r} + \frac{1+\hat{\kappa}}{r}\right)$, where $\hat{\kappa} \equiv -(1 + \boldsymbol{\sigma} \cdot \boldsymbol{L})$. Then Eq. (6) reduces to

$$\begin{aligned}(\gamma^0\partial^0 + i\,m)(m\,\Psi^r) &+ \left(\frac{\partial}{\partial r} + \frac{2-\hat{\kappa}}{r}\right)(m\,\gamma^r\Psi^r) \\ &- \frac{1}{3}\left(\gamma^0\gamma^r\partial^0 - \frac{1-\hat{\kappa}}{r}\right)(m\,\Phi) - \frac{m}{r}\gamma^0\left(\Psi^0 - \frac{1}{3}\gamma^0\Phi\right) = 0,\end{aligned} \tag{7}$$

where $\Psi^r \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{\Psi}$.

In order to solve these equations, we seek normal-mode solutions for $\Psi^\mu(\boldsymbol{r}, t)$ in their most general form consistent with conservation of angular momentum. For the time-like component, we have

$$\Psi^0(\boldsymbol{r}, t) = \mathrm{e}^{-iEt}\,\frac{1}{r}\begin{pmatrix} G_0(r)\,i\,Y_\ell^j(\hat{\boldsymbol{r}}) \\ -F_0(r)\,Y_{\ell'}^j(\hat{\boldsymbol{r}}) \end{pmatrix}, \tag{8}$$

where $Y_\ell^j(\hat{\boldsymbol{r}}) \equiv [Y_\ell(\hat{\boldsymbol{r}}) \otimes \chi]^j$, χ is the spin-$\frac{1}{2}$ spinor, and $\ell' \equiv 2j - \ell$ is the other value of ℓ possible for the given value of the angular momentum quantum number j. For the space-like component, we have the superposition

$$\boldsymbol{\Psi}(\boldsymbol{r},t) = \mathrm{e}^{-iEt}\,\frac{1}{r}\begin{pmatrix} G_1(r)\,\boldsymbol{Y}^j_{\ell_1\frac{3}{2}}(\hat{\boldsymbol{r}}) + G_2(r)\,\boldsymbol{Y}^j_{\ell_2\frac{3}{2}}(\hat{\boldsymbol{r}}) + G_3(r)\,\boldsymbol{Y}^j_{\ell'\frac{1}{2}}(\hat{\boldsymbol{r}}) \\ F_1(r)\,i\,\boldsymbol{Y}^j_{\tilde{\ell}_1\frac{3}{2}}(\hat{\boldsymbol{r}}) + F_2(r)\,i\,\boldsymbol{Y}^j_{\tilde{\ell}_2\frac{3}{2}}(\hat{\boldsymbol{r}}) + F_3(r)\,i\,\boldsymbol{Y}^j_{\ell\frac{1}{2}}(\hat{\boldsymbol{r}}) \end{pmatrix}, \tag{9}$$

where $\boldsymbol{Y}^j_{\ell s}(\hat{\boldsymbol{r}}) \equiv [Y_\ell(\hat{\boldsymbol{r}}) \otimes [\chi \otimes \boldsymbol{e}_1]_s]^j$, with $\boldsymbol{e}_1$ the unit vector. The values of $\ell_1, \ell_2, \tilde{\ell}_1$ and $\tilde{\ell}_2$ in Eq. (9) are constrained by angular momentum considerations ($\boldsymbol{\ell}_i = \boldsymbol{j} + \boldsymbol{\frac{3}{2}}$), and by the Rarita-Schwinger equations themselves, as will be seen shortly.

Before trying to solve Eqs. (5) and (7), we first consider several identities for a vector field field $\boldsymbol{V}^j_{\ell s}(\boldsymbol{r}) \equiv f_\ell(r)\boldsymbol{Y}^j_{\ell s}(\hat{\boldsymbol{r}})$. These are

$$\hat{\boldsymbol{r}} \cdot \boldsymbol{V}^j_{\ell s}(\boldsymbol{r}) = \sum_{\ell'} \alpha^{\ell'}_{j\ell s}\, Y^j_{\ell'}(\hat{\boldsymbol{r}})\, f_\ell(r), \tag{10a}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{V}^j_{\ell s}(\boldsymbol{r}) = \sqrt{3}\, Y^j_\ell(\hat{\boldsymbol{r}})\, \delta_{s,\frac{1}{2}}\; f_\ell(r), \tag{10b}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{V}^j_{\ell s}(\boldsymbol{r}) = \sum_{\ell'} \alpha^{\ell'}_{j\ell s}\, Y^j_{\ell'}(\hat{\boldsymbol{r}}) \left(\frac{d}{dr} + \frac{\beta_{\ell'\ell}}{r}\right) f_\ell(r), \tag{10c}$$

where

$$\beta_{\ell'\ell} \equiv \begin{cases} -\ell, & \text{if } \ell' = \ell + 1; \\ (\ell+1), & \text{if } \ell' = \ell - 1; \end{cases} \tag{11a}$$

$$\alpha^{\ell'}_{j\ell s} \equiv [s][\ell][\ell']\,(-1)^{j+s-1} \begin{pmatrix} \ell & 1 & \ell' \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \ell & 1 & \ell' \\ \frac{1}{2} & j & s \end{Bmatrix}, \tag{11b}$$

with $[j] \equiv \sqrt{2j+1}$. It then follows from putting Eqs. (8) and (9) into Eq. (4) that

$$\Phi = \mathrm{e}^{-iEt}\,\frac{1}{r}\begin{pmatrix} (G_0(r) - \sqrt{3}F_3(r))\, i\, Y^j_\ell \\ (F_0(r) + \sqrt{3}G_3(r))\, Y^j_{\ell'} \end{pmatrix}. \tag{12}$$

Hence Eq. (5) reduces to the set

$$(2E - m)G_0(r) - m\sqrt{3}F_3(r) + \frac{2}{3}\left(\frac{d}{dr} - \frac{\kappa}{r}\right)(2F_0(r) - \sqrt{3}G_3(r)) = 0, \tag{13a}$$

$$-(2E + m)F_0(r) + m\sqrt{3}G_3(r) + \frac{2}{3}\left(\frac{d}{dr} + \frac{\kappa}{r}\right)(2G_0(r) + \sqrt{3}F_3(r)) = 0, \tag{13b}$$

with $\kappa = -(\ell+1)$ if $j = \ell + \frac{1}{2}$, and $\kappa = \ell$ if $j = \ell - \frac{1}{2}$.

For the other three sets of equations, it is useful to define the following quantities:

$$\tilde{G}_1(r) \equiv \tfrac{1}{2}\begin{bmatrix} \sqrt{\frac{2j-1}{j}} \\ \sqrt{\frac{2j-1}{3(j+1)}} \end{bmatrix} G_1(r), \qquad \tilde{G}_2(r) \equiv \tfrac{1}{2}\begin{bmatrix} \sqrt{\frac{2j+3}{3j}} \\ \sqrt{\frac{2j+3}{(j+1)}} \end{bmatrix} G_2(r),$$

$$\tilde{F}_1(r) \equiv \tfrac{1}{2}\begin{bmatrix} \sqrt{\frac{2j-1}{3(j+1)}} \\ \sqrt{\frac{2j-1}{3(j+1)}} \end{bmatrix} F_1(r), \qquad \tilde{F}_2(r) \equiv \tfrac{1}{2}\begin{bmatrix} \sqrt{\frac{2j+3}{j+1}} \\ \sqrt{\frac{2j+3}{3j}} \end{bmatrix} F_2(r), \tag{14}$$

$$\alpha \equiv \begin{bmatrix} -(j-\frac{1}{2}) \\ -(j+\frac{1}{2}) \end{bmatrix}, \qquad \alpha' \equiv \begin{bmatrix} -(j+\frac{1}{2}) \\ -(j-\frac{1}{2}) \end{bmatrix}, \qquad \kappa \equiv \begin{bmatrix} -(j+\frac{1}{2}) \\ (j+\frac{1}{2}) \end{bmatrix},$$

where the upper (lower) component applies for $j = \ell + \frac{1}{2}$ ($j = \ell - \frac{1}{2}$). We also introduce the following four functions:

$$\tilde{G}(r) \equiv \tilde{G}_1(r) - \tilde{G}_2(r) - \frac{1}{\sqrt{3}}G_3(r), \tag{15a}$$

$$\tilde{F}(r) \equiv \tilde{F}_1(r) - \tilde{F}_2(r) - \frac{1}{\sqrt{3}}F_3(r), \tag{15b}$$

$$\tilde{G}'(r) \equiv \alpha\tilde{G}_1(r) - (1-\alpha)\tilde{G}_2(r) + \frac{\kappa}{\sqrt{3}}G_3(r), \tag{15c}$$

$$\tilde{F}'(r) \equiv \alpha'\tilde{F}_1(r) - (1-\alpha')\tilde{F}_2(r) - \frac{\kappa}{\sqrt{3}}F_3(r). \tag{15d}$$

Then Eq. (3), together with (4), implies that $\Phi = \frac{2i}{m^2}\frac{dm}{dr}\Psi^r$, and so we find

$$G_0(r) - \sqrt{3}F_3(r) = \frac{2}{m^2}\frac{dm}{dr}\tilde{G}(r), \tag{16a}$$

$$F_0(r) + \sqrt{3}G_3(r) = -\frac{2}{m^2}\frac{dm}{dr}\tilde{F}(r). \tag{16b}$$

The angular momentum constraints for $\ell = j - \frac{1}{2}$ require $\ell' = j + \frac{1}{2}$, $\ell_1 = j - \frac{3}{2}$, $\ell_2 = j + \frac{1}{2}$, $\tilde{\ell}_1 = j - \frac{1}{2}$ and $\tilde{\ell}_2 = j + \frac{3}{2}$. Similarly, the constraints for $\ell = j + \frac{1}{2}$ require $\ell' = j - \frac{1}{2}$, $\ell_1 = j - \frac{1}{2}$, $\ell_2 = j + \frac{3}{2}$, $\tilde{\ell}_1 = j - \frac{3}{2}$ and $\tilde{\ell}_2 = j + \frac{1}{2}$. Eq. (4) reduces to

$$(2E+m)G_0(r) - m\sqrt{3}F_3(r) + 2\frac{d\tilde{G}(r)}{dr} + 2\frac{\tilde{G}'(r)}{r} = 0, \tag{17a}$$

$$(2E-m)F_0(r) - m\sqrt{3}G_3(r) + 2\frac{d\tilde{F}(r)}{dr} + 2\frac{\tilde{F}'(r)}{r} = 0. \tag{17b}$$

Finally, Eq. (7) implies that

$$
\begin{aligned}
&-(E-m)(m\tilde{G}(r))-\frac{m}{3r}(2G_0(r)+\sqrt{3}F_3(r))-\frac{1}{3}E\,m(F_0(r)+\sqrt{3}G_3(r))\\
&\qquad+\frac{m}{3r}(1-\kappa)(G_0(r)-\sqrt{3}F_3(r))-\left(\frac{d}{dr}+\frac{1-\kappa}{r}\right)(m\tilde{F}(r))=0, \qquad \text{(18a)}\\
&-(E+m)(m\tilde{F}(r))-\frac{m}{3r}(2F_0(r)-\sqrt{3}G_3(r))+\frac{1}{3}E\,m(G_0(r)-\sqrt{3}F_3(r))\\
&\qquad+\frac{m}{3r}(1+\kappa)(F_0(r)+\sqrt{3}G_3(r))+\left(\frac{d}{dr}+\frac{1+\kappa}{r}\right)(m\tilde{G}(r))=0. \qquad \text{(18b)}
\end{aligned}
$$

Thus we have derived eight equations (13, 16–18), in eight unknown functions, that together constitute the Rarita-Schwinger equations in a finite, spherically symmetric system. Eqs. (16) can be used to eliminate $G_3(r)$ and $F_3(r)$, while Eqs. (17) give $\tilde{G}'(r)$ and $\tilde{F}'(r)$ in terms of $G_0(r)$, $F_0(r)$, $\tilde{G}(r)$, and $\tilde{F}(r)$. We are then left with four linear, first-order differential equations involving only four functions. This system of equations (with energy eigenvalue E) can be readily solved using standard numerical techniques.

III. PROBABILITY DENSITY

We now turn our attention to the probability density for the system. The conserved current J^μ corresponding to the Lagrangian $\mathcal{L}_{RS}$ is given by

$$J^\mu=\overline{\Psi}_\alpha\gamma^\mu\Psi^\alpha-\frac{1}{3}\left(\Phi^\dagger\gamma^0\Psi^\mu+\overline{\Psi}^\mu\Phi\right)+\frac{1}{3}\Phi^\dagger\gamma^0\gamma^\mu\Phi. \qquad (19)$$

Explicitly, we have that $\partial\cdot J=0$, if Ψ^μ satisfies the Euler-Lagrange equations. Then $\rho\equiv J^0=\rho_0-\mathbf{\Psi}^\dagger\cdot\mathbf{\Psi}$ can be considered to define the probability density of the spin-$\frac{3}{2}$ particle, where

$$
\begin{aligned}
\rho_0&=\Psi^{0\dagger}\Psi^0-\frac{1}{3}(\Phi^\dagger\gamma^0\Psi^0+\Psi^{0\dagger}\gamma^0\Phi)+\frac{1}{3}\Phi^\dagger\Phi\\
&=\frac{1}{r^2}|Y^j_\ell|^2\left\{\frac{2}{3}G_0^2(r)+F_3^2(r)\right\}+\frac{1}{r^2}|Y^j_{\ell'}|^2\left\{\frac{2}{3}F_0^2(r)+G_3^2(r)\right\}, \qquad (20)
\end{aligned}
$$

using the explicit forms for Ψ^0 and Φ. The quantity $\mathbf{\Psi}^\dagger\cdot\mathbf{\Psi}$ can be evaluated from Eq. (9), leading to

$$
\begin{aligned}
\rho(r) = \frac{1}{4\pi r^2}[j]^2 \sum_{\mathcal{L}\mathrm{even}} [\mathcal{L}]^2 \begin{pmatrix} j & j & \mathcal{L} \\ m_j & -m_j & 0 \end{pmatrix} P_{\mathcal{L}}(\cos\theta)(-1)^{m_j+1/2} \\
\times \Bigg[[\ell]^2 \begin{Bmatrix} \mathcal{L} & j & j \\ \frac{1}{2} & \ell & \ell \end{Bmatrix} \begin{pmatrix} \ell & \ell & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} \frac{2}{3} G_0^2(r) + [\ell']^2 \begin{Bmatrix} \mathcal{L} & j & j \\ \frac{1}{2} & \ell' & \ell' \end{Bmatrix} \begin{pmatrix} \ell' & \ell' & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} \frac{2}{3} F_0^2(r) \\
+ \sum_{i,k=1}^{2} \Bigg(G_i(r)\, G_k(r)\, [\ell_i][\ell_k] \begin{Bmatrix} \mathcal{L} & j & j \\ \frac{3}{2} & \ell_i & \ell_k \end{Bmatrix} \begin{pmatrix} \ell_i & \ell_k & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} \\
+ F_i(r)\, F_k(r)\, [\tilde{\ell}_i][\tilde{\ell}_k] \begin{Bmatrix} \mathcal{L} & j & j \\ \frac{3}{2} & \ell_i & \ell_k \end{Bmatrix} \begin{pmatrix} \tilde{\ell}_i & \tilde{\ell}_k & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} \Bigg) \Bigg], \qquad (21)
\end{aligned}
$$

$$
= \frac{1}{4\pi r^2} \left\{ \frac{2}{3} G_0^2(r) - G_1^2(r) - G_2^2(r) + \frac{2}{3} F_0^2(r) - F_1^2(r) - F_2^2(r) \right\} + \text{higher-}\mathcal{L}\text{ terms.} \qquad (22)
$$

For $j = \frac{1}{2}$, only the above $\mathcal{L} = 0$ term is allowed. Similarly, only this term contributes for $j = \frac{3}{2}$ at $\cos\theta = \pm\frac{1}{\sqrt{3}}$.

Explicit numerical solution of the Rarita-Schwinger equations, Eqs. (13) and (18), as well as the dependent Eqs. (16) and (17), for $j = \frac{1}{2}$ ($\ell = 0, 1$) and $j = \frac{3}{2}$ ($\ell = 1, 2$) were undertaken for several physically plausible potentials. In particular, we examined the solutions for a particle of mass 1230 MeV in potentials of nuclear range (a few fm.) and depth (a few hundred MeV), corresponding to the Δ in a nuclear environment. Although reasonable values of the energy eigenvalue were found, the wave functions typically caused the term in brace brackets in Eq. (22) to change sign as r varies. This corresponds to the existence of a negative probability density, since only the overall sign of J^μ lacks any physical significance. This indicates either that the interpretation of $\rho(r)$ as being a probability density is invalid, or that the Rarita-Schwinger formalism with a scalar potential is intrinsically unphysical. In addition, our attempts to use the two degrees of freedom inherent in the more general form for $\mathcal{L}_{RS}$ to see if another choice of the parameters could lead to a $\rho(r)$ that is explicitly positive definite were also unsuccessful. All parameter sets led to the same qualitative behavior as was observed for the Lagrangian of Eq. (1).

The lack of a positive definite probability density suggests that many of the problems that plague attempts to quantize spin-$\frac{3}{2}$ fields are also to be found in the classical field equations.

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